

# Gradient Based Image Completion by Solving Poisson Equation

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**Abstract.** Image completion is a method to fill the missing portions of an image caused by the removal of one or more foreground or background elements. In this paper a novel image completion algorithm is proposed for removing significant objects from natural images or photographs. The completion is realized in the following three steps. First, a gradient-based model is presented to determine the gradient-patch filling order. This step is critical because a better filling order can improve the continuation of image structures. Second, we implement the gradient-patch update strategy by measuring the exponential distance between the source patch and the target one in gradient domain. In order to find a better patch matching and propagating algorithm, we incorporate the gradient and color information together to determine the target patch. Third, a complete image is achieved by solving the Poisson equation with the updated image gradient map. Some experimental results on real-scene photographs are given to demonstrate both the efficiency and image equality of our novel method.

## 1 Introduction

The removal of objects or large objects, known as image completion, has become an important task in photo editing or film post-production. Given an input image  $I$  with an missing or unknown region, the goal of image completion is to propagate structure and texture information from the known or existing regions  $I - \Omega$  to  $\Omega$ . The removed parts can be filled in by various interactive procedures such as clone brush strokes and compositing processes. However, filling the background hole seamlessly and automatically from existing neighborhood information of damaged images is still a difficult problem. A professional skilled artist usually completes them by meticulous work.

A number of approaches related to image completion have been proposed in computer graphics and computer vision literatures, [4, 5, 6, 9, 16]. Bertalmio [4] used PDE-based method to repair damaged images. The idea is to extend the structures inward arriving at the boundaries of the damaged area. For an image in which only small portions are missing, this approach can achieve highly

smoothed results. However, the lack of texture in a large reconstructed area is easily visible. Therefore this approach is ineffective for filling in large holes in the natural images. Levin [2] extended the idea by measuring global image statistics based on the prior image knowledge besides the local color information. Drori et.al. [6] incorporated pyramid image approximation and adaptive image fragments together to achieve impressive results. However, the method is very slow due to the high computational complexity.

Recently, some researchers have considered example-based method as a way to achieve image region completion with objects or large objects removal [3, 5, 6, 9, 16]. One of the first attempts to use example-based synthesis for object removal was by Harrison [8], which filled the pixels in the target region by the level of "texturedness" of the pixel's neighborhood. Although the intuition sounds, strong linear structures were often overruled by nearby noise. Jia [9] presented a technique for filling image regions through explicitly segmenting the unknown area into different homogeneous texture areas using tensor voting method, but their approach requires both an expensive segmentation step and a difficult choice about what constitutes a boundary between two textures. More recently Criminisi et.al. [5] addressed a example-based image inpainting algorithm with region filling, who used the angle between the isophote direction and the normal direction of the local boundary to define the searching order of the patches, so that the structure of the missing region can be filled before filling in the texture. Sun [19] introduced a novel structure propagation approach to image completion. In their system, the user manually specifies important missing structure information by extending a few curves or line segments from the known to the unknown regions. Their approach synthesizes image patches along these user-specified curves in the unknown region using patches selected around the curves in the known region. Structure propagation is formulated as a global optimization problem by enforcing structure and consistency constraints. If only a single curve is specified, structure propagation is solved using Dynamic Programming. When multiple intersecting curves are specified, the Belief Propagation algorithm is adopted to find the optimal patches. After completing structure propagation, the remaining unknown regions are filled using patch-based texture synthesis.

Our gradient domain techniques and Poisson equation solving techniques also relates to those used for high dynamic range compression [7], Poisson image editing [11], image fusion for context enhancement [13], interactive photomontages [1], Poisson image matting [14] and removing photography artifacts [18]. In our approach we propagate the missing image components by selecting the most similar gradient of the target region with that of the source regions, then the missing background components of the damaged image can be restored with the inpainted gradient map by directly solving the Poisson equation.

The rest of the paper is arranged as follows. Section 2 describes the details of our gradient-based completion mechanism in three key stages. Section 3 shows the results and compares with other previous schemes. In Section 4, we conclude our proposed algorithm with a discussion on future directions.

## 2 Gradient-Based Image Completion Algorithm

In this section, we describe an outline of our image completion algorithm. A digital photograph is firstly used as the input image which containing a manually masked area as the unknown region, then the unknown region is filled in gradient domain by the image information in known area.

We use similar notation as that used in the image completion literature [4, 5, 6, 16]. We denote the unknown region (the region to be filled) by  $\Omega$ , the known region by  $\Psi$ , the contour region by  $\partial\Omega$ , the source and target gradient patches by  $\Psi_s, \Psi_t$  respectively. At each step, a target fragment is firstly completed by adding more details to it from a source fragment consists of gradients, using the gradient-based patch priorities defined in our paper to determine the patch filling order. Secondly, a source gradient patch is selected by measuring the adjusted appearance of the source patch with the target patch, enforcing the searching area in the neighborhood around the previous source patch in the gradient domain. Thirdly, we solve Poisson equation with the updated gradient map to reconstruct a complete image. The experimental results show our algorithm is none-blurring and well structure maintaining.

### 2.1 Gradient-Based Filling Order

Features describing image content, such as color histogram, gradient, texture, shape and object composition, are usually introduced to extract image salient information. These features are then used to determine the filling order of target patches. Zhang et.al. [16] incorporated the textureiness in the neighborhood for determining the filling order, and Criminisi et.al [5] used the angle between the isophote direction and the normal direction of the local boundary to define the searching order of the patches, so that the structure of the missing region can be filled before filling in texture. In this paper we calculate the patch filling order using image gradient feature in a manner similar to that in [5].

The confidence term  $C(p)$  is initialized to zero if  $p$  is removing or damaged; otherwise it is set to one [5], where  $p$  is the pixel under consideration. The second relevant term is called the gradient term  $G(p)$ , which corresponds to the local shape feature, and its value is based on the magnitude of the gradient information at location  $p$ . The gradient term is computed as follows:

$$G(p) = \frac{1}{|A|} \sum_{p_i \in A} \sqrt{Gx^2(p_i) + Gy^2(p_i)} \tag{1}$$

$$C(p) = \begin{cases} 0, & \forall p \in \Omega \\ 1, & \forall p \in I - \Omega \end{cases} \tag{2}$$

where  $A$  denotes the neighborhood area around pixel  $p$ , and  $G = [Gx, Gy]$  denotes the gradient field of an image for the horizontal and vertical directions using a simple forward difference.

For a given patch  $\Psi_s$  centered at the point  $p$  for some  $p \in \partial\Omega$ , where  $p$  is the pixel under consideration, we define its filling order as the following formula:

$$P(p) = C(p) \cdot G(p) \tag{3}$$

$$C(p) = \frac{\sum_{q \in \Psi_s \cap (I \setminus \Omega)} C(q)}{|\Psi_p|} \tag{4}$$

The gradient term  $G(p)$  may be considered as a measure of the amount of edge and structure information surrounding the pixel  $p$ . The purpose for calculating the patch priority value  $P(p)$  is to encourage the linear structures to be filled first with the larger values, therefore, this can help to propagate the broken lines into the connected ones.

## 2.2 Patch Matching and Propagating

After all patch filling priorities on the filling-boundary have been computed, the gradient patch  $\Psi_T$  with the highest priority is firstly selected to propagate. Then the target region is filled with extracted data based on the source region  $\Psi_S$ . As noted before, under the assumption that the content in the unknown area is similar to the content of the known region for the similarity measurement [16], the traditional inpainting techniques propagate pixel-information via diffusion [4, 6], which results in blurry fill-in and line un-continuous, especially of large regions. Criminisi [5] propagated the filling patches by direct sampling of the source region. Similarly, we solve this problem by gradient-patch sampling and gradient-patch copying algorithm. However, we do not use the common Sum of Squared Difference (SSD), which is widely used in image completion to measure the similarity between space patches. The reason is that the SSD does not always suffice to provide the desired completion results as described in [15].

Since a well-suited similarity measurement between gradient patches is the heart of the algorithm that directly influences the final completion result, we use an exponential similarity measure as follows:

$$s(\Psi_S, \Psi_T) = e^{d_c(\Psi_S, \Psi_T) + d_g(\Psi_S, \Psi_T)} \tag{5}$$

$$d_c(\Psi_S, \Psi_T) = \sum_{(x,y)} \|\Psi_S^c(x, y) - \Psi_T^c(x, y)\| \tag{6}$$

$$d_g(\Psi_S, \Psi_T) = \sum_{(x,y)} \|\Psi_S^g(x, y) - \Psi_T^g(x, y)\| \tag{7}$$

$$\Psi_S = \underset{\Psi_i \in \Phi}{\operatorname{argmin}} \frac{s(\Psi_S, \Psi_T)}{|\Psi_S|} \tag{8}$$

where  $\Psi_S^c, \Psi_T^c$  represent the  $R, G, B$  color information of the source patch and the target one, while  $\Psi_S^g, \Psi_T^g$  represent the corresponding gradient information. Once a gradient patch to the highest priority location  $p$  is copied, we then fill it with data extracted from the known region  $\Psi$ . The confidence at all previously damaged pixels in  $\Psi_S$  is updated using formula (4).

As described in the above sections, now we have gotten the final updated gradient map  $G' = [Gx', Gy']$  of the source image for the following restoration through Poisson equation solver.

### 2.3 Image Reconstruction by Solving Poisson Equation

Image reconstruction from gradients fields is an approximate invariability problem, and it is a very active research area. In 2D, a modified gradient vector field  $G' = [Gx', Gy']$  may not be integrable. Let  $I'$  denote the completion image propagated from  $G'$ , we can use one of the direct methods recently proposed [7] to minimize  $|\nabla I' - G|$ , so that  $G = \nabla^2 I'$ . Involving a Laplacian and a divergence operator,  $I'$  can be obtained by solving the Poisson differential equation:

$$\nabla^2 I' = \text{div}([Gx', Gy']) \quad (9)$$

Since both the Laplacian  $\nabla^2$  and  $\text{div}$  are linear operators, approximating them using standard finite differences yields a large system of linear equations. We use the full multigrid method [12] to solve the Laplacian equation with Gaussian-Seidel smoothing iterations. This leads to  $O(n)$  operations to reach an approximate solution, where  $n$  is the number of pixels in the image.

To solve the Poisson equation more efficiently, an alternative is to use a "rapid Poisson solver", which uses a sine transform based on the method [12] to invert the Laplacian operator. However, the complexity with this approach will be  $O(n(\log(n)))$ . The images were zero-padded on both sides, and Dirichlet boundary conditions instead of Neumann boundary conditions were used to avoid the scale/shift ambiguity [18] in the gradient restoration.

## 3 Experimental Results and Discussions

Our algorithm has been applied to a variety of full color natural photographs with complex background structures. Since visual perceptual completion is the ability of the visual system to fill in missing areas [6], it is commonly accepted that the quality of the results apparently corresponds to the human perception of the appearance in the completed images. Our experimental results visually demonstrate that the proposed algorithm can get satisfactory image completion. Moreover, we compared our results with the ones of earlier work.

Figure 1 demonstrates the advantage of our gradient-based patch compensation to match the target patch. This image is downloaded from the website <http://www.cis.rit.edu/fairchild/personal.php>. Even the removing region (the foreground person) covers about 39%, our method can still restore the missing background reasonably.

Figures 2, 3 show comparisons of the results obtained by our gradient-based method with the ones obtained by other proposed methods. In both cases, our method performs better than the previous techniques designed for the restoration of small scratches. For the example shown in Figure 3, where larger objects are removed, our approach outperforms earlier work dramatically in terms of perceptual quality. In figures 2, 3, we can find the blur introduced by the diffusion process and the lack of texture in the synthesized area with the previous methods described [4, 6]. The images obtained by our approach provide more detailed and coherent results than the other ones.



**Fig. 1.** Algorithm comparisons. Top left: original image. Top right: the figure needs to be removed (in white with red boundary). First middle left: the initial gradient map in horizontal direction. First middle right: the initial gradient one in vertical direction. Second middle left: the gradient map after propagating in horizontal direction. Second middle right: the gradient one after propagating in vertical direction. Bottom left: results obtained by inpainting method [4]. Bottom right: results obtained by our method.

When comparing the performance of a new completion algorithm with the previous ones, the subjective image quality evaluation testing, which is based on many observers that evaluate image quality, is not enough. We need an objective image quality testing based on mathematical calculations. The objective quality



**Fig. 2.** Algorithm comparisons. Top left: original image. Top right: the microphone needs to be removed. Middle left: results obtained by inpainting method [4,6]. Middle right: results obtained by our method. Bottom left: the enlarged area from the marked region in top-right image. Bottom right: the enlarged area from the marked region in bottom-left image.

evaluation is easier and faster than the subjective one because no observers are needed. In this paper, we utilize the Peak Signal Noise Ratio (PSNR), which is the most widely used objective image quality metrics, to evaluate the complete image results in red, green, blue channels separately. Table 1 shows our new algorithm has better performance in PSNR than previous algorithms.

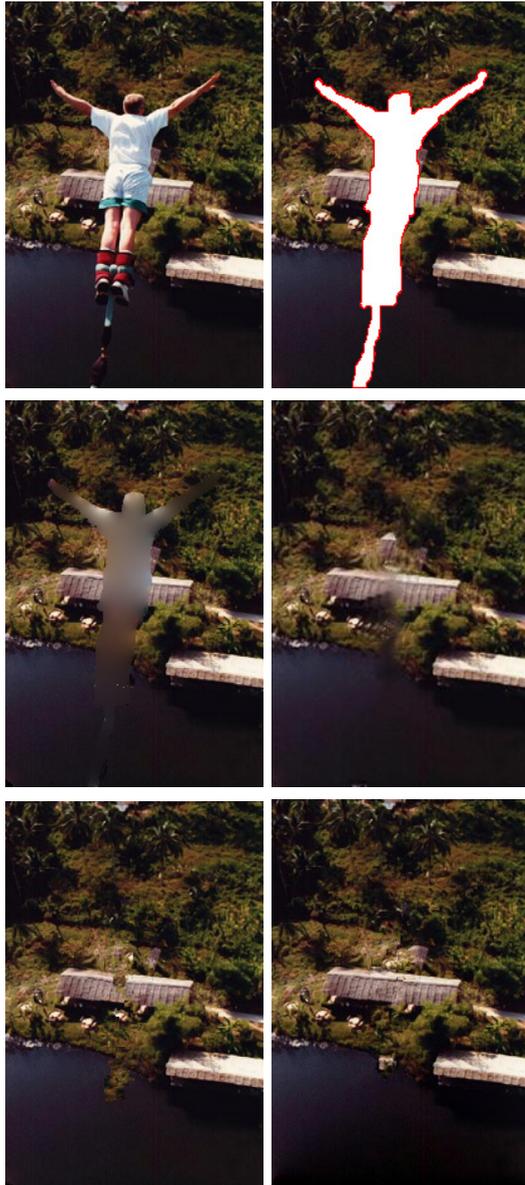
The performance of our method is dependent on the availability of the similar content in the known area. In case of no available patches in the known area to synthesize the unknown one, our algorithm may not work well. Our algorithm also has limitation when dealing with curved structures in still photographs.

**Table 1.** PSNR (dB) comparison between different completion algorithms

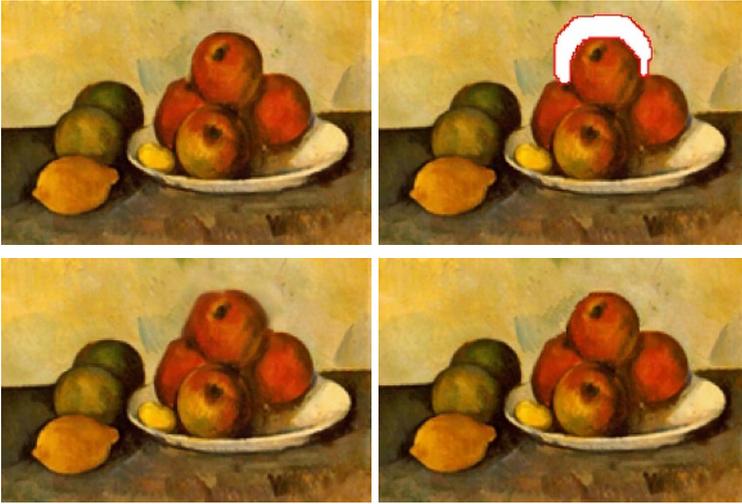
Channel	Algorithm	Fig.1	Fig.2	Fig.3
Red	Bertalmio [4]	11.0962	32.2705	16.8124
	Drori [6]	11.4835	32.4725	17.1017
	Criminisi [5]	11.9430	32.8189	17.4278
	Our Method	12.5741	33.6307	19.3317
Green	Bertalmio [4]	12.0853	32.9155	16.9019
	Drori [6]	12.4321	33.5248	17.1027
	Criminisi [5]	12.6542	33.8171	17.6515
	Our Method	13.1995	34.4735	19.1372
Blue	Bertalmio [4]	11.1060	33.6374	16.0834
	Drori [6]	11.3595	34.1269	16.3772
	Criminisi [5]	11.6643	34.6080	16.5796
	Our Method	12.0598	35.3863	18.2548
Mask(pixels)		78824	3235	7997

Fig.4 shows the results of our gradient-based algorithm on one of the examples used from [6]. As it can be observed by comparing the result from [6] and our completion result, our algorithm does not introduce the edge blur and the ghost artifact. Our result is similar to or slightly better than Drori's [6].

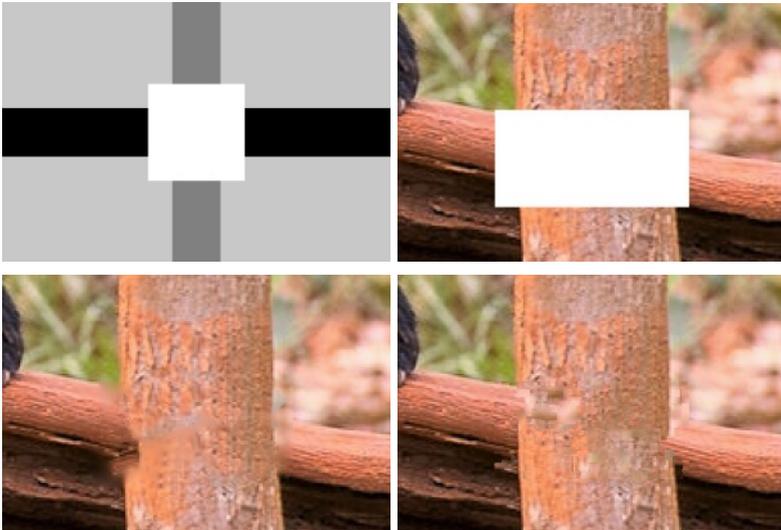
Some limitations remain in our approach. The gradient-based method works well if the missing structures can be represented by a set of linear structures. Our approach also shares the most common limitation of example-based techniques [4, 5, 6, 9, 19]: if there are not enough samples in the image, it will be impossible to synthesize the desired structure or texture. Our approach has no ability to handle depth ambiguities, where the missing area covers the intersection of two perpendicular regions as shown in Fig.5. In our algorithm, pixel colors are represented in RGB color space, we may achieve the better experimental results in the CIE lab color space because of its property of perceptual uniformity and its more meaningful similarity distances than in RGB color space [21, 5].



**Fig. 3.** Algorithm comparison. Top left: original image from [4]. Top right: the target region covers 13% of the total image area. Middle left: the result of region filling by traditional image inpainting [4]. Middle right: result from [6]. Bottom left: the result image by example-based completion [5]. Bottom Right: the final image where the bungee jumper has been completely removed and the occluded region reconstructed by our automatic algorithm.



**Fig. 4.** Algorithm comparison. Top left: A photo of the oil painting "Still Life with Apples", P. Cezanne, c. 1890, The Hermitage, St. Petersburg. Top right: The manually selected target region. Bottom left: results obtained by [6]. Bottom right: results obtained by our gradient method. Notice the ghost artifact disappears since no smoothing is introduced at any stage of our algorithm.



**Fig. 5.** Our approach does not handle depth ambiguities. Top left: A synthesis image in which the missing area covers the intersection of two perpendicular regions [6]. Top right: a nature image with the same ambiguity. Bottom left: results obtained by [6]. Bottom right: results obtained by our method.

## 4 Conclusions and Future Work

A novel gradient-based image completion algorithm by solving Poisson equation has been proposed in this paper. Experiments and analysis both demonstrates the feasibility and efficiency of our new proposed algorithm.

Our image completion approach can be divided into three major steps. First, a gradient-based model is presented to determine the gradient-patch filling order. Second, the gradient-patch update strategy is implemented by measuring the exponential distance of the source patch with the target one in gradient domain. In order to find a better patch matching and propagating algorithm, we incorporate the gradient and color information together to determine the target patch. Third, a complete image is achieved by solving the Poisson equation with the updated image gradient map.

Currently, we are intending to extend our approach from still photography completion to video gradient completion and meshes gradient completion [20]. The difficulties in removing objects from video include global motion compensation and maintaining consistency of the unknown area over the whole video sequence [15, 17].

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